

P - V criticality of charged dilatonic black holes

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In this paper, we investigate the critical behavior of charged black holes of Einstein-Maxwell-dilaton gravity in the presence of two Liouville-type potentials which make the solution asymptotically neither flat nor AdS and has a parameter Λ treated as a thermodynamic quantity that can vary. We obtain a Smarr-type relation for charged dilatonic black holes and find out that the volume is different from the geometrical volume. We study the analogy of the Van der Waals liquid-gas system with the charged dilatonic black hole system while we treat the black hole charge as a fixed external parameter. Moreover, we show that the critical values for pressure, temperature and volume are physical provided the coupling constant of dilaton gravity is less than one and the horizon is sphere. Finally, we calculate the critical exponents and show that they are universal and are independent of the details of the system although the thermodynamic quantities depend on the dilaton parameter and the dimension of the spacetime.

I. INTRODUCTION

In general, the cosmological constant Λ is treated as a fixed parameter when one considers the thermodynamic properties of a black hole. But based on some basic theories, which state that physical constants are not fixed a priori and may arise as vacuum expectation values, the cosmological constant may be treated as a variable parameter. Variation of the cosmological constant affects the thermodynamic behaviours of gravitational systems and extends the phase space of the system [1–5]. Indeed, thermodynamic properties of black holes in anti de Sitter (AdS) space are improved in an extended phase space in which the cosmological constant and its conjugate variable are considered as thermodynamic pressure and volume, respectively. For instance, the study of thermodynamic properties of charged AdS black holes shows that a first-order phase transition between large and small black holes occurs which is analogous to the Van der Waals liquid-gas phase transition [6]. This analogy can be improved by extending the thermodynamic phase space in which the same physical quantities are compared with the liquid-gas system [7]. For instance, in nonextended phase space we find that the coexistence line is in the $\beta - Q$ plane while in the extended phase space, as in the liquid-gas system, the transition occurs in the $P - T$ plane. The analogy between a charged black hole and Van der Waals system has been generalized to the higher dimensional charged black holes, rotating black holes, charged rotating black holes

and Born-Infeld black holes in AdS space [8, 9]. Also some relevant discussions in the extended phase space can be found in Refs. [10], while this subject in Lovelock and $f(R)$ gravity have been investigated in Ref. [11].

In this paper we investigate the critical behavior of $(n + 1)$ -dimensional topological charged dilatonic black holes in the extended phase space with fixed charge parameter (canonical ensemble). The word topological is used for the black holes whose horizons have topology other than sphere such as toroid or hyperbola. The presence of the dilaton field in Einstein-Maxwell theory changes the causal structure of the spacetime and affects on the thermodynamic properties of the black holes. Due to this fact, it is worth investigating the effects of dilaton on the the analogy of dilatonic charged black hole as a Van der Waals system. In an extended phase space, the mass of black holes does not determine the internal energy but it is related to the enthalpy, which includes a contribution from the energy formation of the system. The thermodynamic volume V can then be deduced in terms of the variables of the black hole spacetime in question which is not necessarily a geometrical volume defined in the spacetime. This occurs for rotating black holes [3], Taub-NUT and Taub-Bolt spacetimes and especially for the extreme Taub-NUT black hole which has a thermodynamic volume with no geometrical candidate at all [12]. Similar to these cases, we find that the volume for the cases of dilatonic charged black holes depends on the coupling constant of dilaton and electromagnetic fields and it is different from the geometrical volume defined in the spacetime.

The outline of this paper is as follows: In Sec. II, we review the thermodynamics of $(n + 1)$ -dimensional topological black hole solutions of Einstein-Maxwell-dilaton theory with two Liouville-type potentials. In Sec. III, obtaining the thermodynamic volume and its conjugate quantity and using the equation of state for charged dilatonic black holes, we investigate the critical behavior of the system and compare it with those of a Van der Waals fluid. Also we study the behavior of the system near the critical point and find the critical exponents. We finish our paper with some concluding remarks.

II. TOPOLOGICAL CHARGED DILATONIC BLACK HOLES

In this section, we review the thermodynamics of $(n + 1)$ -dimensional topological charged black holes in dilaton gravity. The action of Einstein-Maxwell theory in the presence of a dilaton field is

[13]:

$$I = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left(\mathcal{R} - \frac{4}{n-1} (\nabla\Phi)^2 - V(\Phi) - e^{-4\alpha\Phi/(n-1)} F_{\mu\nu} F^{\mu\nu} \right) \quad (1)$$

where Φ is the dilaton field, $F_{\mu\nu}$ is the Maxwell field strength defined as $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ with vector potential A and α is the coupling constant of the scalar and electromagnetic fields. In order to have topological dilatonic black holes, $V(\Phi)$ in Eq. (1) should be chosen as [14]

$$V(\Phi) = 2\Lambda e^{4\alpha\Phi/(n-1)} + \frac{k(n-1)(n-2)\alpha^2}{b^2(\alpha^2-1)} e^{4\Phi/[(n-1)\alpha]}, \quad (2)$$

where Λ is a parameter which will be treated as a thermodynamic quantity.

The action (1) with potential (2) admits a static black hole solution with metric [14]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 R(r)^2 d\Omega_k^2, \quad (3)$$

where $d\Omega_k^2$ stands for the line element of an $(n-1)$ -dimensional maximal symmetric space with constant curvature $(n-1)(n-2)k$. We denote the area of this $(n-1)$ -dimensional surface with unit radius by ω_{n-1} . Considering the field equations of Einstein–Maxwell–dilaton gravity [14] with metric (3) and using $R(r) = e^{2\alpha\Phi/(n-1)}$, one can show that the metric function $f(r)$, the dilaton field $\Phi(r)$ and the electromagnetic field $F_{\mu\nu}$ are given as [14]

$$f(r) = \frac{2\Lambda(\alpha^2+1)^2 b^{2\gamma}}{(n-1)(\alpha^2-n)} r^{2(1-\gamma)} - \frac{k(n-2)(\alpha^2+1)^2 b^{-2\gamma} r^{2\gamma}}{(\alpha^2-1)(\alpha^2+n-2)} - \frac{m}{r^{(n-1)(1-\gamma)-1}} + \frac{2q^2(\alpha^2+1)^2 b^{-2(n-2)\gamma}}{(n-1)(\alpha^2+n-2)} r^{2(n-2)(\gamma-1)} \quad (4)$$

$$\Phi(r) = \frac{(n-1)\alpha}{2(1+\alpha^2)} \ln\left(\frac{b}{r}\right), \quad F_{tr} = \frac{q e^{4\alpha\Phi/(n-1)}}{(rR)^{n-1}} \quad (5)$$

where b is a nonzero positive arbitrary constant, $\gamma = \alpha^2/(\alpha^2+1)$ and the parameters m and q are mass and charge parameters, respectively. The mass parameter m can be written in term of the horizon radius as [14]

$$m(r_+) = \frac{2\Lambda(\alpha^2+1)^2 b^{2\gamma}}{(n-1)(\alpha^2-n)} r_+^{n(1-\gamma)-\gamma} - \frac{k(n-2)(\alpha^2+1)^2 b^{-2\gamma}}{(\alpha^2-1)(n+\alpha^2-2)} r_+^{n-2+\gamma(3-n)} + \frac{2q^2(\alpha^2+1)^2 b^{-2\gamma(n-2)}}{(n-1)(n+\alpha^2-2)} r_+^{(n-3)(\gamma-1)-1}, \quad (6)$$

where r_+ denotes the radius of the event horizon which is the largest root of $f(r_+) = 0$. The ADM mass and the electric charge of the black hole are [14]

$$M = \frac{b^{(n-1)\gamma}(n-1)\omega_{n-1}}{16\pi(\alpha^2+1)}m, \quad Q = \frac{q\omega_{n-1}}{4\pi}. \quad (7)$$

The Hawking temperature of the topological black hole on outer horizon r_+ can be calculated as [14]

$$\begin{aligned} \mathcal{T} = \frac{f'(r_+)}{4\pi} = & -\frac{k(n-2)(\alpha^2+1)b^{-2\gamma}}{4\pi(\alpha^2-1)}r_+^{2\gamma-1} \\ & -\frac{\Lambda(\alpha^2+1)b^{2\gamma}}{2\pi(n-1)}r_+^{1-2\gamma} - \frac{q^2(\alpha^2+1)b^{-2(n-2)\gamma}}{2\pi(n-1)}r_+^{(2n-3)(\gamma-1)-\gamma}. \end{aligned} \quad (8)$$

Using the so called area law of the entropy in Einstein gravity which states that the entropy of the black hole is a quarter of the event horizon area [15], one obtains [14]

$$S = \frac{b^{(n-1)\gamma}r_+^{(n-1)(1-\gamma)}\omega_{n-1}}{4} \quad (9)$$

The electric potential U , measured at infinity with respect to the horizon, is defined by

$$U = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r=r_+}, \quad (10)$$

where $\chi = \partial_t$ is the null generator of the horizon. Since the gauge potential A_t corresponding to the electromagnetic field (5) can be written as

$$\begin{aligned} A_t &= \frac{qb^{(3-n)\gamma}}{\Upsilon r^\Upsilon}, \\ \Upsilon &= (n-3)(1-\gamma) + 1, \end{aligned} \quad (11)$$

the electric potential is [14]

$$U = \frac{qb^{(3-n)\gamma}}{\Upsilon r_+^\Upsilon}. \quad (12)$$

It is worth mentioning that the $k=1$ black holes are thermally unstable. Indeed, as one can see in Ref. [14], a Hawking-Page phase transition can happen for these black holes.

III. PHASE TRANSITION OF CHARGED DILATONIC BLACK HOLES IN $(n+1)$ DIMENSIONS

Here, we investigate the thermodynamics of $(n+1)$ -dimensional charged dilatonic black holes in an extended phase space, treating the cosmological constant and its conjugate quantity as thermodynamic variables associated with the pressure and volume, respectively. Since we are

going to discuss the thermodynamics of the black hole in the extended phase space by introducing the pressure proportional to the cosmological constant, the black hole mass M should be considered as the enthalpy $H \equiv M$ rather than the internal energy of the gravitational system [2]. Using the fact that the entropy of black hole is a quarter of the area of the horizon, the thermodynamic volume $V = \int 4S dr_+$ is obtained as

$$V = \frac{b^{(n-1)\gamma} \omega_{n-1}}{\varepsilon} r_+^\varepsilon, \quad (13)$$

$$\varepsilon = (n-1)(1-\gamma) + 1 = \frac{n+\alpha^2}{1+\alpha^2}, \quad (14)$$

which is different from the geometrical volume. Using the first law of thermodynamics

$$dM = \mathcal{T}dS + UdQ + Vd\mathcal{P}, \quad (15)$$

one can show that the pressure, which is the conjugate quantity of the thermodynamic volume, is

$$\mathcal{P} = -\frac{[n-\gamma(n-1)]b^{2\gamma}}{8\pi[n-\gamma(n+1)]r_+^{2\gamma}}\Lambda = -\frac{(n+\alpha^2)b^{2\gamma}}{8\pi(n-\alpha^2)r_+^{2\gamma}}\Lambda. \quad (16)$$

One may note that the above \mathcal{P} is proportional to the cosmological constant Λ and reduces to the pressure for Reissner-Nordstrum black hole in the absence of dilaton ($\gamma = 0$). Also, one should note that the pressure is positive provided $\alpha^2 < n$ ($\gamma < n/(n+1)$). The above thermodynamic quantities satisfy the following Smarr formula:

$$M = \frac{(n-1)(1-\gamma)}{\Upsilon} \mathcal{T}S + UQ + \frac{(4\gamma-2)}{\Upsilon} V\mathcal{P}. \quad (17)$$

Now, we study the analogy of the liquid-gas phase transition of the Van der Waals fluid with the phase transition of the charged dilatonic black hole system in the extended phase space in canonical ensemble, in which we treat the black hole charge Q as a fixed external parameter.

A. Equation of state

Using Eqs. (8) and (16) for a fixed charge Q , one may obtain the equation of state $\mathcal{P}(V, \mathcal{T})$ as

$$P = \frac{(n+\alpha^2)(n-1)T}{4(n-\alpha^2)(1+\alpha^2)r_+} + \frac{k(n-2)(n-1)(n+\alpha^2)b^{-2\gamma}}{16\pi(\alpha^2-1)(n-\alpha^2)r_+^{2-2\gamma}} + \frac{(n+\alpha^2)q^2b^{-2(n-2)\gamma}}{8\pi(n-\alpha^2)r_+^{1-(2n-3)(\gamma-1)+\gamma}} \quad (18)$$

where r_+ is a function of the thermodynamic volume V given in Eq. (13). Before we proceed further, we perform the dimensional analysis to translate the ‘geometric’ equation of state (18) to a physical one. The physical pressure and temperature are given by

$$\mathcal{P} = \frac{\hbar c}{l_p^2} P, \quad \mathcal{T} = \frac{\hbar c}{\kappa} T, \quad (19)$$

where the Planck length reads $l_p^2 = \hbar G/c^3$ and κ is the Boltzmann constant. Therefore

$$\mathcal{P} = \frac{(n + \alpha^2)(n - 1)\kappa\mathcal{T}}{4l_p^2(n - \alpha^2)(1 + \alpha^2)r_+} + \frac{k(n - 2)(n - 1)(n + \alpha^2)\hbar cb^{-2\gamma}}{l_p^2 16\pi(\alpha^2 - 1)(n - \alpha^2)r_+^{2-2\gamma}} + \frac{(n + \alpha^2)q^2\hbar cb^{-2(n-2)\gamma}}{l_p^2 8\pi(n - \alpha^2)r_+^{1-(2n-3)(\gamma-1)+\gamma}}. \quad (20)$$

Comparing with the Van der Waals equation [7], we conclude that one should identify the specific volume v of the fluid with the horizon radius of black hole as

$$v = \frac{4l_p^2(1 + \alpha^2)(n - \alpha^2)r_+}{(n - 1)(n + \alpha^2)} \quad (21)$$

Using the above identification and returning to geometric units, the equation of state (18) can be written as

$$P = \frac{T}{v} + \frac{k(n - 2)(n - \alpha^2)^{1-2\gamma}b^{-2\gamma}}{4^{2\gamma}\pi(\alpha^2 - 1)(\alpha^2 + 1)^{2\gamma-2}[(n + \alpha^2)(n - 1)]^{1-2\gamma}v^{2-2\gamma}} + \frac{q^2b^{-2(n-2)\gamma}(n + \alpha^2)^{(2n-3)(\gamma-1)-\gamma}(n - 1)^{(2n-3)(\gamma-1)-\gamma-1}}{2^{(4n-6)(\gamma-1)-2\gamma+1}\pi(n - \alpha^2)^{(2n-3)(\gamma-1)-\gamma}(1 + \alpha^2)^{(2n-3)(\gamma-1)-\gamma-1}v^{\gamma-(2n-3)(\gamma-1)+1}}. \quad (22)$$

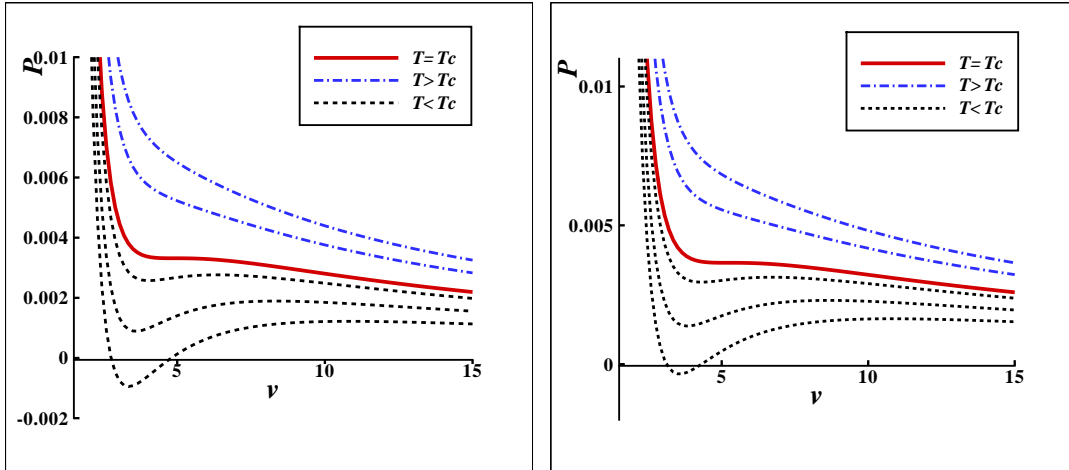


FIG. 1: $P - v$ diagram of charged dilatonic black hole in the case of $k = 1$ for $n = 3$, $q = b = 1.0$, $\alpha = 0$ (left) and $\alpha = 0.3$ (right).

Now, we are ready to investigate the possibility of critical behavior for the case of charged dilatonic black holes. To do this, we calculate the pressure and volume at the critical point which are known as the critical pressure and the critical volume. This can be done by solving the following equations:

$$\left. \frac{\partial P}{\partial v} \right|_{T_c} = 0, \quad \left. \frac{\partial^2 P}{\partial v^2} \right|_{T_c} = 0. \quad (23)$$

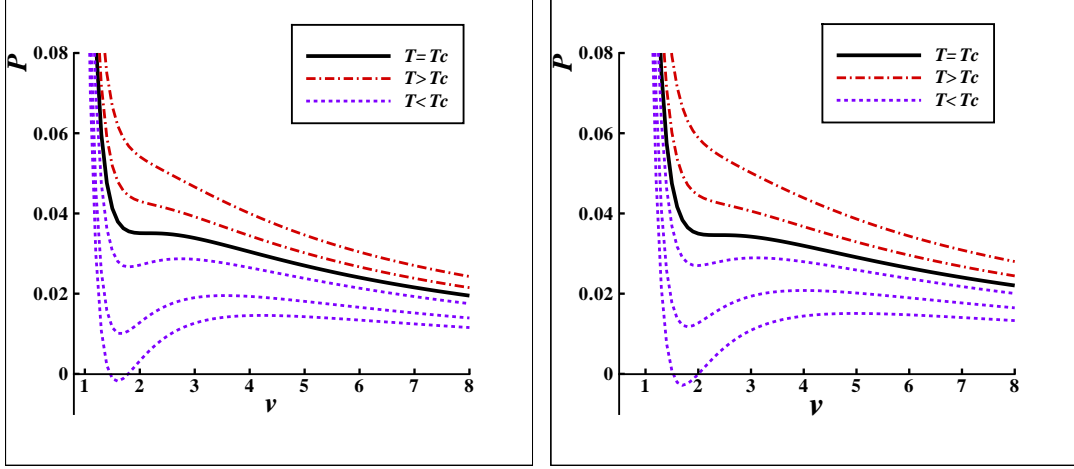


FIG. 2: $P - v$ diagram of charged dilatonic black hole in the case of $k = 1$ for $n = 4$, $q = b = 1.0$, $\alpha = 0.3$ (left) and $\alpha = 0.5$ (right).

One obtains

$$v_c = \left[\frac{2q^2(n-1+\alpha^2)(2n-3+\alpha^2)b^{6\gamma-2n\gamma}}{k(n-2)(n-1)} \right]^{1/2\Upsilon} \times \left[\frac{4(n-\alpha^2)(1+\alpha^2)}{(n-1)(n+\alpha^2)} \right] \quad (24)$$

$$T_c = \left[\frac{(n-1)}{2q^2(n-1+\alpha^2)} \right]^{(1-2\gamma)/2\Upsilon} \times \left[\frac{(2n-3+\alpha^2)}{k(n-2)} \right]^{[(2n-3)(\gamma-1)-\gamma]/2\Upsilon} \times \left[\frac{(\alpha^2+n-2)b^{-\gamma(n-1)/\Upsilon}}{\pi(1-\alpha^2)} \right] \quad (25)$$

$$P_c = \left[\frac{k(n-1)(n-2)}{(2n-3+\alpha^2)(2n-2+2\alpha^2)} \right]^{[\gamma-(2n-3)(\gamma-1)+1]/2\Upsilon} \times \left[\frac{(n+\alpha^2)(2n-3+\alpha^2)(n-2+\alpha^2)b^{-2\gamma/\Upsilon}}{8\pi(1+\alpha^2)(n-\alpha^2)q^{2(1-\gamma)/\Upsilon}} \right] \quad (26)$$

This gives the following universal ratio

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{(1-\alpha^2)(2n-3+\alpha^2)}{4(n-1+\alpha^2)} \quad (27)$$

Note that for $\alpha = 0$ in four-dimensional spacetime, we recover the ratio $\rho_c = 3/8$ which is the characteristic of Van der Waals fluid. It is worth mentioning that a positive value for universal ratio ρ_c is guaranteed provided $\alpha < 1$. In this case, Eq. (22) shows that only for the case of $k = 1$, the critical behavior occurs. This is due to the fact that for the cases of $k = 0$ and $k = -1$ with $\alpha < 1$, all the terms in Eq. (22) are positive and therefore no critical behavior will occur. The $P - v$ isothermal diagrams of the case $k = 1$ for different values of dilaton coupling constant $\alpha < 1$ in 4 and 5 dimensions are shown in Figs. 1 and 2. As in the case of Van der Waals gas, there is a critical point which is a point of inflection on the critical isotherm. Obviously, for $T < T_c$ there is a small/large black hole phase transition in the system.

B. Gibbs free energy

Thermodynamic behavior of a system may be described by its thermodynamic potential, which is the free energy in canonical ensemble. But, since we are considering an extended phase space with variable cosmological constant, we associate it with the Gibbs free energy $G = M - TS$ [5]. It is a matter of calculations to show that the the Gibbs free energy reduces to

$$G = G(T, P) = \left\{ \frac{k(n-2)(n-2+\alpha^2)b^{(n-3)\gamma}}{16\pi(1+\alpha^2)r_+^{(1-n)(1-\gamma)+(1-2\gamma)}} + \frac{P(\alpha^2-1)(\alpha^2+1)b^{(n-1)\gamma}}{(n-1)(n+\alpha^2)r_+^{n(\gamma-1)-\gamma}} \right. \\ \left. + \frac{q^2(2n-3+\alpha^2)(\alpha^2+1)b^{(3-n)\gamma}}{8\pi(n-2+\alpha^2)(n-1)r_+^{(n-2)(1-\gamma)+\gamma}} \right\} \omega_{n-1}, \quad (28)$$

where r_+ should be understood as a function of pressure and temperature via the equation of state (18). The behaviour of the Gibbs free energy is depicted in Fig. 3. This figure demonstrates the characteristic swallowtail behavior and therefore there is a first order phase transition in the system. Using this fact that the Gibbs free energy, temperature and the pressure of the system are constant during the phase transition, one can plot coexistence curves of large ($r_+ = r_l$) and small ($r_+ = r_s$) charged dilatonic black holes. These are shown in Fig. 4.

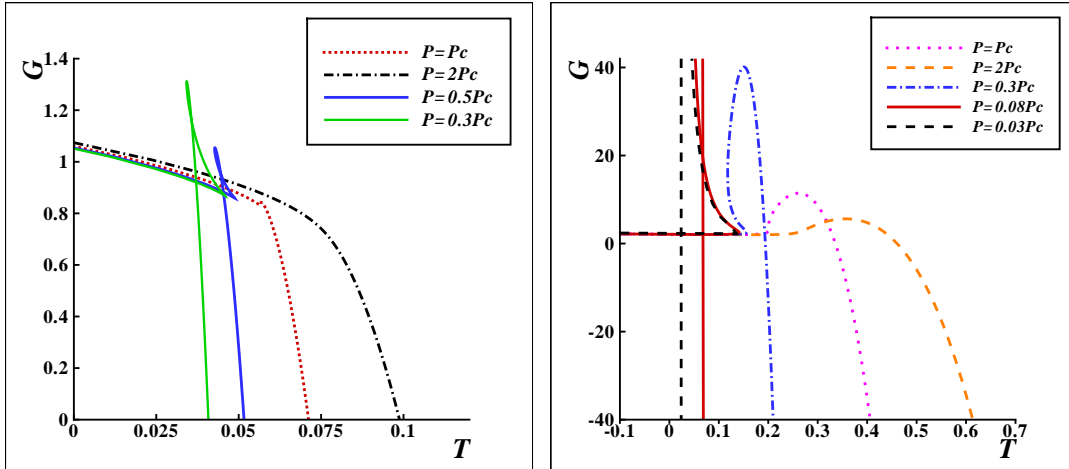


FIG. 3: Gibbs free energy of charged dilatonic black hole with $\alpha = 0.3$, $q = b = 1.0$ for $n = 3$ (left) and $n = 4$ (right).

C. Critical exponents

The critical exponent characterizes the behavior of physical quantities in the vicinity of the critical point. So, following the approach of [8], we calculate the critical exponents α' , β' , γ' and

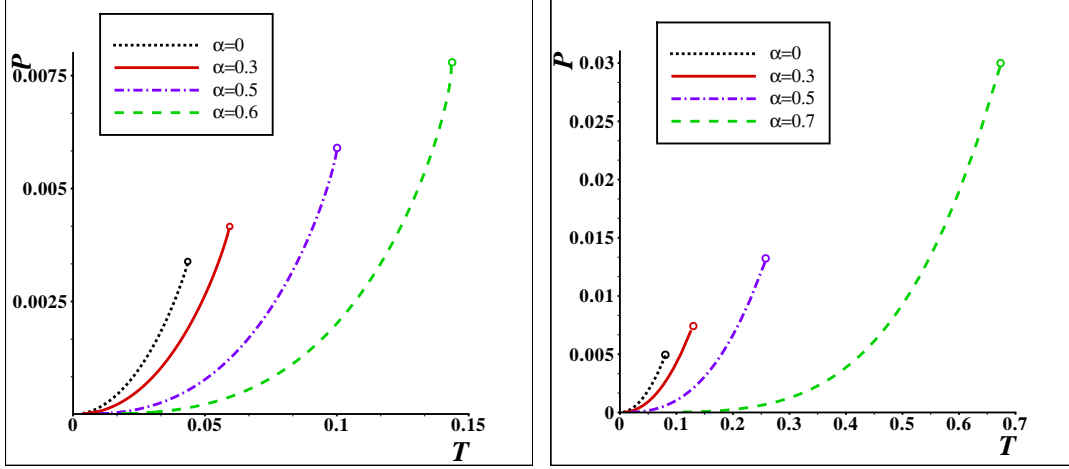


FIG. 4: Coexistence curves of small-large black hole phase transition of charged dilatonic black hole for $q = b = 1.0$ and different values of α with $n = 3$ (left) and $n = 4$ (right) in the P-T plane. The critical points are highlighted by a small circle at the end of the coexistence curves.

δ' for the phase transition of an $(n + 1)$ -dimensional charged dilatonic black hole. To calculate the critical exponent α' , we consider the entropy S (9) as a function of T and V . Using (13) we have

$$S = S(T, V) = \frac{(b^{(n-1)\gamma} \omega_{n-1})^{(1+\alpha^2)/(n+\alpha^2)}}{4} \left(\frac{n + \alpha^2}{1 + \alpha^2} V \right)^{(n-1)/(n+\alpha^2)}, \quad (29)$$

which is independent of T . Since the exponent α' governs the behavior of the specific heat at constant volume $C_V \propto |t|^{\alpha'}$ and

$$C_V = T \left. \frac{\partial S}{\partial T} \right|_V = 0$$

one finds that $\alpha' = 0$.

To obtain the other exponents, we define the reduced thermodynamic variables

$$p \equiv \frac{P}{P_c}, \quad \nu \equiv \frac{v}{v_c}, \quad \tau \equiv \frac{T}{T_c}.$$

So, equation of state (22) translates into ‘the law of corresponding state’:

$$p = \frac{2(2n - 2 + 2\alpha^2)}{(2n - 3 + \alpha^2)(1 - \alpha^2)} \frac{\tau}{\nu} + \frac{(2n - 2 + 2\alpha^2)(1 + \alpha^2)}{(\alpha^2 - 1)(2n - 4 + 2\alpha^2)} \frac{1}{\nu^{2-2\gamma}} + \frac{(1 + \alpha^2)}{(2n - 3 + \alpha^2)(n - 2 + \alpha^2)} \frac{1}{\nu^{\gamma+1+(2n-3)(1-\gamma)}}. \quad (30)$$

In order to find the other critical exponents, we follow the method of Ref. [8] and expand Eq. (30)

near the critical point

$$t = \tau - 1, \quad \omega = \nu^\varepsilon - 1 = \frac{V}{V_c} - 1. \quad (31)$$

One obtains

$$p = 1 + At - Bt\omega - C\omega^3 + O(t\omega^2, \omega^4), \quad (32)$$

where

$$A = \frac{1}{\rho_c} = \frac{4(n-1+\alpha^2)}{(2n-3+\alpha^2)(1-\alpha^2)}, \quad (33)$$

$$B = \frac{1}{\varepsilon\rho_c} = \frac{4(n-1+\alpha^2)(1+\alpha^2)}{(2n-3+\alpha^2)(1-\alpha^2)(n+\alpha^2)}, \quad (34)$$

$$C = \frac{2(n-1+\alpha^2)}{3(1+\alpha^2)^2\varepsilon^3} = \frac{2(n-1+\alpha^2)(1+\alpha^2)}{3(n+\alpha^2)^3}. \quad (35)$$

Denoting the volume of small and large black holes by ω_s and ω_l , respectively, differentiating Eq. (32) with respect to ω at a fixed $t < 0$, and applying the Maxwell's equal area law [7] one obtains

$$\begin{aligned} p &= 1 + At - Bt\omega_l - C\omega_l^3 = 1 + At - Bt\omega_s - C\omega_s^3 \\ 0 &= -P_c \int_{\omega_l}^{\omega_s} \omega (Bt + 3C\omega^2) d\omega, \end{aligned} \quad (36)$$

Equation (36) leads to the unique nontrivial solution

$$\omega_l = -\omega_s = \sqrt{-\frac{Bt}{C}}, \quad (37)$$

which gives the order parameter $\eta = V_c(\omega_l - \omega_s)$ as

$$\eta = 2V_c\omega_l = 2\sqrt{-\frac{B}{C}}t^{1/2}. \quad (38)$$

Thus, the exponent β' which describes the behaviour of the order parameter η near the critical point is $\beta' = 1/2$. To calculate the exponent γ' , we may determine the behavior of the isothermal compressibility near the critical point

$$\kappa_T = -\frac{1}{V} \frac{\partial V}{\partial P} \Big|_T \propto |t|^{-\gamma'}.$$

Since $dV/d\omega = V_c$, the isothermal compressibility near the critical point reduces to

$$\kappa_T = -\frac{1}{V} \frac{\partial V}{\partial P} \Big|_T \propto \frac{V_c}{BP_c t}, \quad (39)$$

which shows that $\gamma' = 1$. Finally the "shape" of the critical isotherm $t = 0$ is given by (32)

$$p - 1 = -C\omega^3, \quad (40)$$

which indicates that $\delta' = 3$.

Although the reduced pressure p in Eq. (30) depends on the dilaton parameter α and the dimension of the spacetime, the critical exponents associated with the charged dilatonic black hole in $(n + 1)$ dimensions are independent of them. This is consistent with the belief that the critical exponents are universal and do not depend on the details of the physical system.

IV. SUMMARY AND CONCLUSIONS

In this paper, we investigated the critical behavior of charged dilatonic black holes of Einstein-Maxwell-dilaton gravity in the presence of the potential $V(\Phi)$ given in Eq. (2) while Λ treated as a thermodynamic quantity that can vary. Using the known thermodynamic quantities, we identified the thermodynamic pressure and volume of the system and obtained the Smarr-type relation for charged dilatonic black holes. As in the cases of rotating [3], Taub-Nut and Taub-bolt black holes [12], we found that the volume is different from the geometrical volume. In canonical ensemble we wrote the equation of state as $P = P(v, T)$ and plot $P - v$ isotherm diagrams. These diagrams are similar to those of Van der Waals fluid and indicate a first-order phase transition between small and large black holes in temperature bellow critical temperature. We found that for any nontrivial value of the charge, there exists a critical temperature T_c below which this phase transition occurs. Moreover, we studied the behavior of certain physical quantities near the critical point and calculated the pressure, temperature and volume at the critical temperature. Although Eq. (23) has solution for the case of $k = -1$ for $\alpha > 1$, critical behavior do not occur in this case. This is due to the fact that the critical ratio $\rho_c = P_c v_c / T_c$ becomes negative for $\alpha > 1$. Thus, we limited ourselves to the case of $k = 1$ and $\alpha < 1$ for which the critical ratio is positive. The characteristic swallowtail behavior of the Gibbs free energy was another indication of the first-order phase transition in the system. Using this fact that the Gibbs free energy, temperature, and pressure of the system are constant during the phase transition, we plotted the coexistence curves of large-small charged dilatonic black holes and again we showed that a first-order phase transition in temperature bellow critical temperature occurs. Finally, we calculated the critical exponents and found that they are universal and are independent of the details of the system although the reduced pressure p in Eq. (30) depends on the dilaton parameter α and the dimension of the spacetime.

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